HOMEWORK 11 - ANSWERS TO (MOST) PROBLEMS

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Section 5.1: Areas and Distances

5.1.2.

(a) (i) $\Delta x = 2$, so

$$L_6 = f(0)(2)+f(2)(2)+f(4)(2)+f(6)(2)+f(8)(2)+f(10)(2) = 18+\frac{52}{3}+\frac{50}{3}+\frac{44}{3}+12+8 = \frac{260}{3} \approx 86.67$$

(ii)

$$R_6 = f(2)(2)+f(4)(2)+f(6)(2)+f(8)(2)+f(10)(2)+f(12)(2) = \frac{52}{3}+\frac{50}{3}+\frac{44}{3}+12+8+2 = \frac{212}{3} \approx 70.67$$

(iii)

$$M_6 = f(1)(2)+f(3)(2)+f(5)(2)+f(7)(2)+f(9)(2)+f(11)(2) = 18+17+15+13+10+\frac{16}{3} = \frac{235}{3} \approx 78.33$$

(b) Overestimate
(c) Underestimate
(d) M_6 (just right, does not overshoot, like L_6, but not undershoot either, like R_6)

5.1.5.

(a) If $n = 3$, then $\Delta x = 1$, and if $n = 6$, $\Delta x = \frac{1}{2}$, so:

$$R_3 = f(0)(1) + f(1)(1) + f(2)(1) = 1 + 2 + 5 = 8$$

$$\begin{aligned} R_6 =& f(-0.5)(0.5) + f(0)(0.5) + f(0.5)(0.5) + f(1)(0.5) + f(1.5)(0.5) + f(2)(0.5) \\ =& 1.25(0.5) + 1(0.5) + 1.25(0.5) + 2(0.5) + 3.25(0.5) + 5(0.5) \\ =& 6.875 \end{aligned}$$

(b)

$$L_3 = f(-1)(1) + f(0)(1) + f(1)(1) = 2 + 1 + 2 = 5$$

$$\begin{aligned} L_6 =& f(-1)(0.5) + f(-0.5)(0.5) + f(0)(0.5) + f(0.5)(0.5) + f(1)(0.5) + f(1.5)(0.5) \\ =& 2(0.5) + 1.25(0.5) + 1(0.5) + 1.25(0.5) + 2(0.5) + 3.25(0.5) \\ =& 5.375 \end{aligned}$$

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(c)

$$M_3 = f(-0.5)(1) + f(0.5)(1) + f(1.5)(1) = 5.75$$

$$M_6 = f(-0.75)(0.5)+f(-0.25)(0.5)+f(0.25)(0.5)+f(0.75)(0.5)+f(1.25)(0.5)+f(1.75)(0.5) = 5.9375$$

(d) M_6

0.1. **5.1.11.** Here $n = 6$ and $\Delta x = 0.5$

$$\begin{aligned} L_6 =& v(0)(0.5) + v(0.5)(0.5) + v(1)(0.5) + v(1.5)(0.5) + v(2)(0.5) + v(2.5)(0.5) \\ =& 0(0.5) + 6.2(0.5) + 10.8(0.5) + 14.9(0.5) + 18.1(0.5) + 19.4(0.5) \\ =& 34.7 \end{aligned}$$

$$\begin{aligned} R_6 =& v(0.5)(0.5) + v(1)(0.5) + v(1.5)(0.5) + v(2)(0.5) + v(2.5)(0.5) + v(3)(0.5) \\ =& 6.2(0.5) + 10.8(0.5) + 14.9(0.5) + 18.1(0.5) + 19.4(0.5) + 20.2(0.5) \\ =& 44.8 \end{aligned}$$

5.1.15. The midpoint sum seems to best approximate the area:

$$M_6 = v(0.5)(1)+v(1.5)(1)+v(2.5)(1)+v(3.5)(1)+v(4.5)(1)+v(5.5)(1) = 50+40+30+18+10+5 = 153ft$$

5.1.19. $\Delta x = \frac{\pi}{2n}$, $x_i = \frac{\pi i}{2n}$, so:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i) = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{\pi}{2n}\right) \frac{\pi i}{2n} \cos(\frac{\pi i}{2n})$$

5.1.21. The area under the curve of $f(x) = \tan(x)$ from 0 to $\frac{\pi}{4}$

Section 5.2: The definite integral

5.2.11. $\Delta x = \frac{1}{5} = 0.2$, so:

$$\begin{aligned} \int_0^1 \sin(x^2)dx \approx& f(0.1)(0.2) + f(0.3)(0.2) + f(0.5)(0.2) + f(0.7)(0.2) + f(0.9)(0.2) \\ =& \sin(0.01)(0.2) + \sin(0.09)(0.2) + \sin(0.25)(0.2) + \sin(0.49)(0.2) + \sin(0.81)(0.2) \\ =& \approx 0.3789 \end{aligned}$$

5.2.18. $\int_{\pi}^{2\pi} \frac{\cos(x)}{x} dx$

5.2.21. Here $\Delta x = \frac{6}{n}$ and $x_i = -1 + \frac{6i}{n}$.

$$\begin{aligned}
\int_{-1}^{5} (1+3x)dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x(1+3x_i) \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}(1+3(-1+\frac{6i}{n})) \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}(-2+\frac{18i}{n}) \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{-12}{n} + \frac{108i}{n^2}) \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{-12}{n} + \sum_{i=1}^{n} \frac{108i}{n^2}) \\
&= \lim_{n\to\infty} \frac{-12}{n}(n) + \frac{108}{n^2} \sum_{i=1}^{n} i) \\
&= \lim_{n\to\infty} -12 + \frac{108}{n^2} \frac{n(n+1)}{2} \\
&= -12 + \frac{108}{2} \\
&= -12 + 54 \\
&= 42
\end{aligned}$$

And you thought 42 was **not** the answer to everything =) !

5.2.23. Here $\Delta x = \frac{2}{n}$ and $x_i = \frac{2i}{n}$

$$\begin{aligned}
\int_{0}^{2} (2-x^2)dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x(2-(x_i)^2) \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}(2-\frac{4i^2}{n^2}) \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} - \frac{8i^2}{n^3} \\
&= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 1 - \frac{8}{n^3} \sum_{i=1}^{n} i^2 \\
&= \lim_{n\to\infty} \frac{4}{n}(n) - \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} \\
&= 4 - \frac{8}{6}2 \\
&= \frac{4}{3}
\end{aligned}$$

5.2.30. $\Delta x = \frac{9}{n}$, so $x_i = 1 + \frac{9i}{n}$, and so:

$$\int_1^{10} x - 4\ln(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} \Delta(x)(x_i - 4\ln(x_i)) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{9}{n}\left(\left(1+\frac{9i}{n}\right) - 4\ln\left(1+\frac{9i}{n}\right)\right)$$

5.2.34.

(a) 4 (the area of the large triangle)
(b) -2π (minus the area of the semicircle)
(c) $4 - 2\pi + \frac{1}{2} = \frac{9}{2} - 2\pi$ (the area of the large triangle minus the area of the semicircle plus the area of the small triangle)

5.2.36. 2π (it's the area of a semicircle with radius 2)

5.2.43. $\int_0^1 5 - 6x^2 dx = 5\int_0^1 1dx - 6\int_0^1 x^2 dx = 5 - 6\frac{1}{3} = 5 - 2 = 3$

5.2.44. $\int_1^3 2e^x - 1dx = 2\int_1^3 e^x dx - \int_1^3 1dx = 2(e^3 - e) - 2$

5.2.47.

$$\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx = \int_{-2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx = \int_{-2}^{5} f(x)dx + \int_{-1}^{-2} f(x)dx = \int_{-1}^{5} f(x)$$

5.2.51. $m \leq f(x) \leq M$, so integrating from 0 to 2, we get $2m \leq \int_0^2 f(x)dx \leq 2M$

5.2.52. On $[0,1]$, $x^2 \leq x$, so $1 + x^2 \leq 1 + x$, so $\sqrt{1+x^2} \leq \sqrt{1+x}$, so integrating from 0 to 1, we get $\int_0^1 \sqrt{1+x^2}dx \leq \int_0^1 \sqrt{1+x}dx$